

CONSTRUCTION PROBLEMS LEADING TO CUBIC EQUATIONS

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ABSTRACT: These equations depict the formulation and solution of problems. The mechanical instrument of Plotemey was later created to address the challenge of doubling a cube.

KEYWORDS: unknown, rational, regular, circular, proportional, angular triceps, circle squaring, irrational, coefficient.

INTRODUCTION

An equation is an analytical expression of a problem in mathematics. It looks for the values of two argument functions that are equal to each other. These arguments, on which the functions depend, are usually called unknowns, and the values that make their functions equal to each other are called solutions to the equation. For example, $3x - 6 = 0$ is an unknown equation, $x = 2$ is its solution; $x^2 + y^2 = 25$ are two unknown equations, $x = 3, y = 4$ is one of its solutions. The number of solutions to a given equation depends on M , the set of values that can be placed in the unknown position. For example, the equation $x^4 - 4 = 0$ has no solution in the set of rational numbers, in the set of real numbers it has two solutions $x_1 = \sqrt{2}, x_2 = -\sqrt{2}$, and in the set of complex numbers four $x_1 = \sqrt{2}, x_2 = -\sqrt{2}, x_3 = i\sqrt{2}, x_4 = -i\sqrt{2}$

The equation $\sin x = 0$ has infinitely many solutions in the set of real numbers:

$$x_k = k\pi \quad (k = 0, \pm 1, \pm 2, \dots)$$

If the unknowns have to satisfy several equations at once, such equations are called *systems* of equations. An unknown algebraic equation is as follows

$$a_0x^n + a_1x^{n-1} + \dots + a_n = 0, \quad (a_0 \neq 0) \quad (1)$$

The degree of the equation is n . The methods for calculating equation (1) when $n = 1$ and $n = 2$ have long been known. There are no solutions formulas for $n = 3$ and $n = 4$. There is at least one

solution to any algebraic equation. The equation $f(x) = 0$ is called a transcendental equation if $f(x)$ is a transcendental function.

Finding approximate solutions to algebraic and transcendental equations, as well as approximate values of issues for differential equations and solutions to integral equations at finite points, is the method of solving equations. The numerical solution of equations entails performing coefficients in the equation and problem conditions, as well as arithmetic operations on specific functions, so that the solution of the equation is found with the required accuracy.

Although the general methods of numerical solution of equations in most problems of mathematics and practice began to be developed only in the 18th century (Newton's method), in the early 13th century the Italian scientist Leonardo Pizansky

$x^3 + 2x + 10x = 20$ had found the roots of the equation $\frac{1}{3}10^{-10}$ with precision. [1].

Koshi, a Samarkand scholar, devised tertiary equations that were solved utilizing the *sequential approach method* in the 15th century. The French mathematician Viet invented a method for computing the roots of algebraic equations in the 16th century.

Arab mathematicians, as well as their European students, have struggled to solve cubic equations. Leonardo of Pisa achieved an outstanding performance in this area.

He showed that the root of the equation $x^3 + 2x^2 + 10x = 20$ is not expressed by the Euclidean irrationality of the form $\sqrt{a + \sqrt{b}}$. It was astonishing that the question of the solution of the radicals, which later matured, was raised for the beginning of the thirteenth century. Mathematicians did not see general ways to solve the cubic equation.

In arithmetic experiments, Gauss uses a compass and a ruler to prove without proof that a regular n -angle cannot be made without n Fermat prime numbers, and that no regular 7 angles can be made in particular. This unfavorable result should not have surprised his contemporaries less than the possibility of making regular 17 corners.

After all, $n = 7$ is the first value for which n cannot be a regular n angle for 7 despite many attempts. Undoubtedly, the Greek geometers doubted that it would be inconvenient to deal with this matter. Archimedes did not propose in vain to make regular 7 corners using conical cuts. However, there is no question of proving that it is impossible.

It should be noted that proving a negative assertion has played an important role in the history of mathematics. Proof of impossibility requires consideration of all thought-provoking solutions: both fabrication and proof. In the case of a positive solution, it is enough to show one specific solution.

Proof of Impossibility in Mathematics The Pythagoreans (sixth century BC) have a famous connection with their desire to bring all mathematics to whole numbers, they buried this idea with their own hands: the square is equal to 2. There is no fraction.

In other words, a square's sides and diagonals don't have a similar dimension. As a result, the total number and its ratios are insufficient to depict even the most basic scenarios. The best minds of Ancient Greece were astounded by this revelation. According to tradition, God punished the Pythagorean who revealed this information to the public (he was killed in a shipwreck). Plato (429-

348 BCE) expresses his surprise at the presence of irrational quantities. Plato was forced to reassess the possibilities of geometry after being confronted with a "practical" dilemma.

Eratosthenes describes the need to build an altar twice as large as before in order for the goddess Delos to escape the plague that afflicted oracle in his Platonic work.

When they turn to Plato for advice, Plato tells them that God did not tell him this prophecy because he needed twice as much sacrifice, but because the Greeks did not think about mathematics and did not appreciate geometry "(Theon Smirnsky).

Plato took advantage of every opportunity to promote science. According to Yevtoni (in response to Glavka's hesitancy over the tombstone), a similar issue arose in one version of Minos' legend.

So we're looking for the side of a double cube, which is the root of the $x^3 = 2$ equation. Eudoxus and Helicon received the delos from Plato. Different solutions were proposed by Menexmus, Architects, and Eudoxus, but none of them could be found using a compass or ruler. In a sonnet etched on a marble block in Ptolemy's palace in Alexandria, Eratosthenes, who later developed a mechanical mechanism to solve the difficulty of doubling a cube, remarked that his predecessors' solutions were exceedingly complex: "You no longer require the smart cylinder of Architecture."

Menexmus did not draw you a triad on a cone, and you do not need the crooked lines of Eudoxus, equated with God...

In solving the equation, he used a conic section. We know nothing about the "crooked lines" of Eudoxus. As for the mechanical solution, Eratosthenes is not the first to write about it. According to Plutarch, "Plato himself rejected his friends Eudoxus, Archytas and Menexum because they wanted to make the duality of the cube mechanical, they did not want to accept the median proportionality with theoretical thinking; for the wealth of geometry disappears, dies, and thus geometry returns to the geometry of contemplation, instead of ascending even higher and standing firmly in eternity, instead of some non-material images that say that God is God.

By the way, Yevtokiy says that some kind of mechanical solution to the problem of delos belongs to Plato himself, which uses carpenter's perforated thought and moving board. Plato is often contrasted with Archimedes (287 - 212 BC), who despises "material things that require long processing by an unworthy profession", which Archimedes used with many of his discoveries, including the defense of Syracuse. famous for its machines.

Along with the Delos problem, Greek mathematics left several problems that could not be created using a compass and a ruler: angle trisection (dividing an angle into 3 equal parts), squaring a circle, and making a regular n angle, specifically a regular 7 angle, 9 make a corner. Some of these problems were related to the cubic equation by Greek mathematicians, and more by Arab mathematicians.

Regular 7 corners $z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ or

$(z^3 + \frac{1}{z^3}) + (z^2 + \frac{1}{z^2}) + (z + \frac{1}{z}) + 1 = 0$ to the equation. $x = z + \frac{1}{z}$ to the variable $x^3 + x^2 - 2x - 1 = 0$ and form an equation.

We show that the roots of the cube doubling and the equations of 7 angles are not quadratic irrationality, which means that they cannot be constructed using a compass and a ruler.

Theorem. If a cube equation with integer coefficient $a_3x^3 + a_2x^2 + a_1x + a_0 = 0$ has a root with quadratic irrationality, then it also has a rational root.

Proof. Let x_1 be such a root. It is formed using arithmetic operations and the square root of an integer. First, the root is obtained from a series of rational numbers $\sqrt{A_1}, \sqrt{A_2}, \dots, \sqrt{A_a}$, then from rational numbers $\sqrt{A_i} (\sqrt{B_1}, \dots, \sqrt{B_b})$ etc. the root is obtained from the numbers formed by arithmetic operations; each step is rooted in some number, which is formed at all previous steps and is expressed arithmetically. A "layer" of quadratic irrationality is formed. Let \sqrt{N} be one of the numbers at the last stage of x_1 formation. Let's focus on how $\sqrt{N} - x_1$. Since $x_1 = \alpha + \beta\sqrt{N}$, then $\sqrt{N} - \alpha - \beta\sqrt{N}$ is not a quadratic irrationality. Suffice it to say that arithmetic operations on expressions of the form $\alpha + \beta\sqrt{N}$ again lead to such expressions: obviously for addition and subtraction, checked directly for multiplication and division.

$$\frac{\alpha + \beta\sqrt{N}}{\gamma + \delta\sqrt{N}} = \frac{(\alpha + \beta\sqrt{N})(\gamma - \delta\sqrt{N})}{\gamma^2 - \delta^2N}$$

The \sqrt{N} in the denominator of the expression must be lost

Now $x_1 = \alpha + \beta\sqrt{N}$ substitute the expression into the equation, and if the operation is completed, then $P + Q\sqrt{N} = 0$ a visual relationship is formed, and in this case P, Q, α, β, a_i are polynomial. If $Q \neq 0$, at that rate $\sqrt{N} = \frac{P}{Q}$ and \sqrt{N} by assigning the value x_1 to the expression, we can create an expression for x_1 that does not contain \sqrt{N} . If $Q = 0$, then it is verified that $x_2 = \alpha - \beta\sqrt{N}$ is also a root, given that $\frac{a_2}{a_3} = x_1 + x_2 + x_3$ is the sum of roots (Vieta's theorem), $x_3 = -\frac{a_2}{a_3} - 2\alpha$, i.e. ., it also has a square irrational root represented by $\sqrt{A_i}, \sqrt{B_i}$ and x_1 , but \sqrt{N} is not involved. Continuing this process, we get rid of all radicals in the expression of the root of the equation layer by layer, starting from the last layer. Then a rational root is formed, the proof is complete.

Now it remains to check whether the equation of interest to us has a rational root. Assume that the leading coefficient of the equation is $a_3 = 1$. Then all rational roots are integer. Let's put in the equation $x = \frac{p}{q}$ (p, q is the reciprocal prime number), multiply both parts by q^3 and make sure that p^3 , as well as p is then divisible by q , i.e. it suffices to make $q = 1$. In this case, if the root, then $a_2 = -\alpha + m, a_1 = -\alpha m + n, a_0 = -\alpha n$, i.e. $m = a_2 + \alpha, n = a_1 + a_2\alpha + \alpha^2$.

So, if a_i and α are integers, then m and n must also be integers, and α is a divisor. As a result, the search for rational roots for the equation $a_3 = 1$ leads to the choice of a finite number of possibilities - to finding the divisors. Integer of the equation we are interested in the absence of a root is easily checked, which means that the absence of a root with square irrationality is also easily checked.

References:

1. O'zbek sovet ensiklopediyasi. 11- tom. Toshkent – 1978. 44 bet.
2. Abduganiyevna K. G. About the Great Uzbek Astronomer and Mathematician, Statesman Mirzo Ulugbek //EUROPEAN JOURNAL OF INNOVATION IN NONFORMAL EDUCATION. – 2022. – Т. 2. – №. 4. – С. 86-88.
3. Вахвалов N.S. “Численные методы” [учеб. пособие], 2 изд., т.1, М., 1975
4. Михлин С.Г., Смолицкий Х.Л., Приближенные методы решения дифференциальных и интегральных уравнений, М., 1965
5. Damirovich, Khamrokulov U., and Turakulova M. Ismatullaevna. "Analysis Of Synchlis Building Calculation Methods." JournalNX, 2020, pp. 53-58.
6. S.G.Gindikin. Fizika va matematiklar haqida hikoyalar. Toshkent “O’qituvchi” 1989. 192-196 b.