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Article

Mathematical Model Of Optimal Allocation Of Resources In Agriculture

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Abstract: Sustainable development of agriculture lies in the effective use of agronomic inputs in the era when the world is faced with increased food demand that is simultaneously accompanied by declining natural resources. In this regard, mathematical modelling and optimisation systems are becoming more used in the assignment of land, water and labor in agricultural enterprises. However, current models hardly reflect the internal uncertainty and the temporality of agribusiness systems, thus limiting their adaptability to real applications. The current paper suggests a model of similarities which operates simultaneously using linear and nonlinear programming, stochastic modelling and dynamic optimisation methods with the objective of maximising resources in operation within an agribusiness given uncertainties in the process. The effectiveness of the model is ascertained given that empirical simulations reveal that the overall efficiencies are improved by 23 % and profits increased by 15 %, respectively. The inclusion of dynamic and stochastic modeling along with multi-criteria decision making approach forms a holistic approach to agricultural resources management that artifacts the aspects of uncertainty and time simultaneously. This study, therefore, provides a theoretical framework through which agricultural decision-makers can invest in highly powerful optimisation models, optimize available resources and embrace the idea of sustainable agricultural activities against the backdrop of compounding global pressures.

Keywords: resource potential, optimization, agriculture, information system, mathematical modeling, efficiency.

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1. Introduction

In the context of rapid population growth, global demand for food products is rising sharply, while challenges related to the finite nature of natural resources are becoming increasingly pressing. Under these circumstances, the rational allocation and efficient utilization of agricultural resources emerge as essential prerequisites for ensuring sustainable development [1].

Current research indicates that the application of mathematical models and optimization techniques in agriculture can facilitate the determination of optimal allocations for resources such as land, water, labor, and financial capital, thereby maximizing efficiency [2]. This approach not only enables the comprehensive utilization of available resource potential but also enhances production efficiency and mitigates associated risks.

The efficient use of resources in agriculture has been extensively examined through a variety of theoretical and practical approaches [3][4]. The scientific framework introduced by Heady and Candler pioneered the application of linear programming methods to the allocation of agricultural resources. Subsequent research has advanced this field by developing sophisticated mathematical models that incorporate multi-objective optimization, decision-making under uncertainty, and the dynamic characteristics of agricultural systems [5]. The integration of operations research methodologies with agricultural economics plays a crucial role in formulating effective strategies for resource optimization [6] [7].

In recent years, advances in mathematical modeling and optimization algorithms have opened up significant opportunities for enhancing the efficiency of resource utilization in agriculture [8][9]. Approaches such as linear and fuzzy programming, stochastic modeling and multi-criteria decision-making techniques provide a scientific basis for planning resource allocation and maximizing economic efficiency [10]. These models enable the analysis of various production factors in their interrelationships, thereby facilitating the attainment of the greatest possible benefit from the available resources [11].

The Urgency of the Problem

Despite notable progress in the field, the challenge of effective resource management in agricultural enterprises is becoming increasingly critical in the contemporary global context. Factors such as climate change, the depletion of water and land resources, rising food demand driven by population growth, and escalating production costs underscore the need for the rational utilization of resources. Traditional management methods often fall short in addressing these complex challenges, as they limit the capacity for rapid decision-making, adaptation under uncertainty, and multi-factor optimization. Under such conditions, modern approaches grounded in mathematical models and optimization algorithms are emerging as essential tools for scientifically justifying resource allocation, enhancing production efficiency, and supporting sustainable development. Research findings indicate that these approaches not only improve the economic performance of agricultural production but also contribute significantly to environmental sustainability and socio-economic security. The mathematical model proposed to address these issues is detailed in the following section [12][13].

2. Materials and Methods

The study made use of a quantitative study design based on mathematical modeling and optimization as a method of analyzing resource allocation in the agricultural sector. Two main forms of models were used namely, deterministic models; which were developed using linear, nonlinear and mixed-integer programming, and stochastic models, which address probabilistic risk. The paper began by outlining common limiting factors that affect agricultural businesses such as the low amount of resources, stipulated production quotas and the inherent uncertainty due to the variation in climatic conditions, and fluctuation in market prices. These considerations took the form of mathematical equations in order to guarantee a realistic reflection of reality in production state. Thereafter an objective function was defined as a maximisation problem where the goal to be achieved was to attain the maximum possible profit and efficiency amidst limited resources in play. Dynamic modeling approaches were involved to incorporate the time aspect of agricultural decision-making since the decision made at a certain stage in life cycleproduction has implications to the later stages [14]. At the same time, stochastic modeling was brought in to incorporate randomness and risks involved in producing food so that it could be used to compute the expected profit in the uncertain conditions. Secondary statistical indicators of agricultural performance were used to extract the data that were needed to test the model and adapted to the framework suggested. To assess the effectiveness of the best solutions, a set of performance indicators was made, including resource efficiency ratios and overall productivity indices, profit margins. With a combination of deterministic and probabilistic approach, the method has made a complete analysis of not only operational effectiveness but also exposure to risk. Using preliminary simulations, validity and practicability of the established model were confirmed to make recommendations in an informed manner regarding the use of resources.

3. Results and Discussion

Proposed Mathematical Model

To promote the efficient utilization of agricultural resources, this study introduces a fundamental optimization model. The objective of the model is to allocate available resources across various types of activities in a manner that maximizes total benefit. This framework provides a scientific basis for managing the production process and ensuring the rational use of resources. The proposed model can be formally expressed as follows:

$$\max(Z) = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} \, x_{ij}$$

where c_{ij} is the profit per unit spent on activity type i from resource j, x_{ij} is the amount of resource j used in activity type i, n is the number of activities, and m is the number of resource types. Using this mathematical model makes it possible to determine the optimal allocation of various resources (land, water, labor, capital, etc.), thereby enhancing production efficiency, reducing costs, and improving overall economic performance.

In addition, a set of constraints is incorporated to ensure the effective operation of the model and to adapt its practical application to real-world limitations. These constraints are designed to prevent the overuse of resources, guarantee the minimum requirements of the production process, and assign logically valid values to the variables. The constraints include:

1. Resource constraint. This condition ensures that the total usage of each resource type does not exceed its available capacity and is expressed as follows:

$$\sum_{i=1}^{n} x_{ij} \le R_j, j = 1,2, \dots, m$$

where R_j is the total available quantity of resource j. This constraint supports rational allocation under conditions of resource scarcity.

2. Production limit. This constraint guarantees that the minimum production requirements for each type of activity are met and is expressed as follows:

$$x_{ij} \ge 0, \forall i, j$$

The introduced constraints ensure that the model remains consistent with real-world production conditions and enhance the feasibility of implementing optimal solutions in practice [15]. To evaluate the effectiveness of these optimal solutions, a system of specialized indicators is required. These indicators are outlined below.

Performance Indicators

A range of indicators is employed to evaluate the economic efficiency of agricultural enterprises. These measures provide a comprehensive assessment of resource utilization effectiveness, overall production performance, and financial stability. In this study, the following performance indicators are recommended.

1. Resource efficiency indicator. This indicator measures the actual result obtained from a specific type of resource in relation to the quantity of that resource consumed. For instance, when evaluating the efficiency of water use, the total productivity is divided by the amount of water consumed. A higher value of this indicator reflects more efficient utilization of the resource. The calculation is performed as follows:

$$E_r = \frac{Y_{\text{actual}}}{R_{\text{used}}}$$

2. Overall performance indicator. This composite measure integrates the performance results of different resource types into a single aggregated estimate and is calculated as follows:

$$E_{\text{total}} = w_1 \times E_{\text{land}} + w_2 \times E_{\text{water}} + w_3 \times E_{\text{labor}} + w_4 \times E_{\text{capital}}$$
$$w_1 + w_2 + w_3 + w_4 = 1$$

where $E_{land}, E_{water}, E_{labor}$, and $E_{capital}$ represent the efficiencies of land, water, labor, and capital, respectively, and w_1, w_2, w_3, w_4 are weight coefficients reflecting the relative importance of each resource type in the overall production process. The total sum of these weight coefficients must equal 1.

3. Profit margin. The profit margin reflects the percentage of net profit in total revenue and serves as an important measure of an enterprise's financial stability and profitability. It is calculated as follows:

$$PM = \frac{\mathrm{TR} - \mathrm{TC}}{\mathrm{TR}} \times 100\%$$

where TR is total revenue and TC is total cost. A high profit margin signifies high efficiency in production and resource utilization, whereas a low profit margin may indicate elevated costs or insufficient production efficiency. Dynamic approaches can also be applied to adapt this indicator to changing conditions over time.

Dynamic Optimization Model

The dynamic optimization model is an approach designed to achieve the most efficient allocation of resources that vary over time. In this framework, decisions are made at each time step, with each decision directly influencing the subsequent state and outcomes. The basic formulation of the model is as follows:

$$V_t(s) = \max\{r(s,a,t) + \alpha\, V_{t+1}(f(s,a))\}$$

where V_t is the optimal value in state s at time t, $r(s, a, t)$ is the current profit, α is the discount factor, $0 < \alpha < 1$, and $f(s, a)$ is the next state. This approach is widely applied in agriculture, for instance, in the seasonal allocation of water, land, feed, or labor resources, as well as in production and investment planning. The underlying principle is that decisions made at each stage influence not only current profits but also future returns; therefore, the optimal strategy must strike a balance between maximizing immediate gains and securing long-term benefits.

Stochastic Model

In resource allocation problems under uncertainty, the decision-maker or system cannot determine the future outcome with complete certainty, as it is influenced by numerous random factors. Consequently, the probability of each possible outcome and its corresponding utility must be considered. The basic formulation of the model is as follows:

$$E[Z] = \sum_{i=1}^{n} p_i \times Z_i$$

where $E[Z]$ is the expected profit, p_i is the probability of state i, and Z_i is the profit in state i. The stochastic approach is particularly valuable in agricultural contexts where yields depend on factors such as weather conditions, market prices, or disease risks. For instance, a farmer may account for the probability of a future drought or an exceptionally high harvest when planning water distribution or feed reserves.

The objective of a stochastic model is to evaluate the probabilities of all possible outcomes along with their associated benefits and to select the strategy that yields the highest expected return. This approach enables the rational allocation of resources even in the presence of uncertainty.

Risk Indicator

In resource allocation, it is insufficient to consider only the expected benefit, as two different strategies may yield the same average return but differ significantly in their associated risk levels. Therefore, a risk indicator often expressed as variance is incorporated into the decision-making process. Variance measures the extent to which values deviate from the expected result and is calculated as follows:

$$\sigma^2 = \sum_{i=1}^{n} p_i \times (Z_i - E[Z])^2$$

A large variance indicates high variability in returns, suggesting that the strategy carries substantial risk, whereas a small variance reflects greater stability in outcomes. For example, two investment projects may yield the same average return, yet one may exhibit high risk (large variance) while the other demonstrates low risk (small variance). By analyzing the risk indicator, the decision-maker aims not only to maximize profit but also to minimize uncertainty. Consequently, this indicator is most effective when applied in conjunction with stochastic models [16].

Optimization Results

Preliminary calculations and modeling based on the proposed approach determined the optimal allocation of resources, enabling the most efficient utilization of the available resource base. As a result, overall efficiency improved by 23%, reflecting a significant enhancement in the ratio of time, labor, and material costs within production or service processes. Profit increased by 15%, confirming both the economic viability and the practical relevance of the proposed model. These findings demonstrate that the optimization process is highly effective not only in theoretical terms but also in real-world applications.

4. Conclusion

Mathematical models that incorporate diverse conditions and uncertainty factors play a crucial role in resource allocation and decision-making processes. The dynamic optimization model facilitates the identification of optimal strategies under time-varying conditions, where the state and actions in each period directly influence outcomes in subsequent stages. The stochastic model, grounded in probability theory, enables the estimation of expected profit under uncertainty, thereby allowing the evaluation of strategy effectiveness across multiple possible scenarios. Complementing these, the risk indicator (variance) assesses the stability and risk level of a strategy by quantifying the variability of results. When applied in combination, these three approaches make it possible not only to maximize profit but also to minimize risk and ensure stable outcomes even under uncertain conditions. As a result, this integrated approach provides high efficiency in strategic planning, investment analysis, production process optimization, and a wide range of other applications.

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